General equilibrium with externalities and tradable licenses

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Abstract. Within a general equilibrium setting, we consider an economy where externalities arise from the use of common resources.
We show that quantity regulation presents problem of equilibrium existence. Then, we consider a cap and trade system on the consumption of commodities that may originate externalities. Moreover, when permissions to consume these commodities are allocated among consumers and can be costless traded, we get existence of equilibrium. However, equilibrium allocations may be, in general, inefficient. Next, we define different core solutions and we show that if externalities do not appear below a given level of consumption then any equilibrium allocation is in the core and, in particular, it is efficient.

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1 Externalities and quantity regulation

Consumption of some commodities may lead to externalities, that is, the consumption by an individual affects not only her preferences but also the preferences of the others. Since externalities are fundamentally about individual facing "wrong" prices for their actions, they are naturally a general equilibrium issue. In fact, the analysis of the price mechanism within a general equilibrium model may help to a better understanding of externalities explaining how the prices can fail to incorporate "external" effects. Moreover, the presence of externalities is a source of inefficiency and different regulatory solutions have been provided in the literature (quantity regulation, Pigouvian taxes, property rights).

To incorporate an external quantity regulation within a general equilibrium framework, let us consider an exchange economy with n consumers and $\ell + k$ commodities. Each agent $i \in N = \{1, \ldots, n\}$ has endowments $\omega_i \in \mathbb{R}_+^{\ell+k}$ and chooses a commodity bundle in the consumption set $\mathbb{R}_+^{\ell+k}$. Consumption of the commodities $\ell + 1, \ldots, \ell + k$ may originate externalities. Thus, each consumer $i \in N$ has a preference relation represented by an utility function

$$\begin{array}{rccl} \mathcal{V}_i : & \mathbb{R}_+^{(\ell+k)n} & \longrightarrow & \mathbb{R} \\ & x & \longrightarrow & \mathcal{V}_i(x) = \mathcal{V}_i(x_i, \tilde{x}_{-i}) \end{array}$$

where $x_i = (\hat{x}_i, \tilde{x}_i) \in \mathbb{R}_+^{\ell+k}$ and $\tilde{x}_{-i}$ denotes the consumption of the commodities that generate externalities of every agent except individual i.

To consider quantity regulation let us state the regulatory solution of stating caps, that is, it is not allowed to consume more than L_j of the commodity $\ell + j$. Let $L = (L_j, j = 1, \ldots, k)$.

Thus, an allocation $x = (x_1, \ldots, x_n)$ is feasible if

(i) $\sum_{i \in N} x_i \leq \sum_{i \in N} \omega_i$ (*physical feasibility*) and

(ii) $\sum_{i \in N} \tilde{x}_i \leq L$ (*quantity regulation*),

where $\tilde{x}_i$ is the consumption bundle of the commodities $\ell + 1, \ldots, \ell + k$ of individual i.

Let $p = (p_1, \ldots, p_\ell, p_{\ell+1}, \ldots, p_{\ell+k})$ denote a price vector for commodities. For every $p \in \mathbb{R}_+^{\ell+k}$ the budget set of the consumer i is

$$B_i(p) = \left\{ x \in \mathbb{R}_+^{\ell+k} | p \cdot x \leq p \cdot \omega_i \right\}$$

An equilibrium is a price vector p for commodities and a feasible allocation $x = (x_i, i \in N)$, such that the bundle x_i maximizes $\mathcal{V}_i(\cdot, x_{-i})$ on the budget set $B_i(p)$ for every i.

A non-existence example. Consider an economy with two consumers (1 and 2) and two commodities (x and y). Each agent is endowed with 2 units of every commodity. The preferences are represented by the following utility functions:

$$\mathcal{V}_1((x_1, y_1), (x_2, y_2)) = \begin{cases} x_1 y_1 & \text{if } \ y_1 + y_2 \leq 2.5 \\ x_1 y_1 - (y_1 + y_2 - 2.5) & \text{otherwise} \end{cases}$$

$$\mathcal{V}_2((x_1, y_1), (x_2, y_2)) = \ x_2 y_2.$$

Let us consider the cap $L = 2.5$ for the commodity y. Note that any allocation which distributes the total endowment of y is not feasible.

We remark that if (p_x, p_y) is an equilibrium price system then $p_y = 0$. This is so because otherwise the budget constraint for either agent 1 or 2 is not binding and then such an agent could increase the consumption of x and becomes better off. The fact that $p_y = 0$ leads to both agents to consume 2 units of commodity x and the utility function of agent i is increasing in y_i, with $i = 1, 2$. That is, none of the agents can be maximizing their preference relations on the budget set if $y_1 + y_2 \leq 2.5$. We conclude that there is no equilibrium. This is basically due to the quantity regulation by the cap L that becomes effective since the total endowment of commodity y is large than the cap.

2 Externalities and tradable rights for consumption

As we have remarked the consumption of some commodities by an individual may produce externalities in the sense that preferences of other consumers are also affected. For instance, the private use of a parking have influence on the parking spaces and therefore at some levels of utilization may affect the preferences of all the potential users. This is also illustrated by some common resources which involves the problem of an overuse.

Let us assume that there is not only a limit to the use or consumption of the commodities involving externalities but also an amount of tradable licenses or rights required to use or consume those commodities. These licenses are shared among the consumers.

To be precise, consider the previous exchange economy with n consumers and $\ell+k$ commodities, where consumption of commodities $\ell+1, ..., \ell+k$ may originate negative externalities. Each agent $i \in N = \{1, \ldots, n\}$ has endowments $\omega_i = (\hat{\omega}_i, \tilde{\omega}_i) \in \mathbb{R}_+^{\ell+k}$ and chooses a commodity bundle $x_i = (\hat{x}_i, \tilde{x}_i)$ in the consumption set $\mathbb{R}_+^{\ell+k}$. We denote by x_i^h the consumption of commodity h by agent i. Hence, both $x_i^{\ell+j}$ and $\tilde{x}_i^j$ denote the $\ell+j$ commodity consumption of individual i. In the economy $\mathcal{E}$ we consider now, consumption of each commodity $\ell+j$ originates externalities when the aggregate consumption get over a level, namely, L_j. Let $L = (L_1, \ldots, L_k)$.

Thus, each consumer $i \in N$ has a preference relation represented by an utility function $\mathcal{V}_i : \mathbb{R}_+^{(\ell+k)n} \to \mathbb{R}$ which is given by

$$\mathcal{V}_i(x) = \begin{cases} U_i(x_i) \text{ if } \sum_{i\in N} \tilde{x}_i \leq L \\ \mathcal{U}_i(x_i, \tilde{x}_{-i}) \text{ otherwise,} \end{cases}$$

where $\tilde{x}_{-i}$ denotes the consumption of the $\ell+1, \ldots \ell+k$ commodities of every agent except individual i.

To tackle the externality problem rights for consumption are established in this economy $\mathcal{E}$. To be precise, in order to get the bundle $z \in \mathbb{R}_+^{\ell+k}$ a vector $f(z)$ of rights are required. That is, f_j states the rights that are necessary to consume or use the commodity $\ell+j$. Note that, in particular, we can consider that f depends only on the consumption of the commodities generating externalities. However, the fact that f depends on the complete commodity bundle allows us to consider a larger variety of situations. To illustrate this point, note that if we consider the use of a parking the dimensions of vehicles become important (for example, the rights may differ for cars and trucks); for the case of a lake, the material used for fishing might determine the amount of rights (the utilization of a fishing rod is not the same as a fishing net).

There are total R of rights and each consumer $i \in N$ is endowed with $r_i \in \mathbb{R}_+^k$ rights or licenses. Thus, $R = \sum_{i\in N} r_i$.

An allocation $x = (x_1, \ldots, x_n)$ is feasible if

(i) $\sum_{i\in N} x_i \leq \sum_{i\in N} \omega_i$ (*physical feasibility*) and

(ii) $\sum_{i\in N} f(x_i) \leq R$ (*rights feasibility*).

We state the following hypotheses:

(A.1) For every agent $i \in N$, the utility function $\mathcal{V}_i$ is a continuous function and $\mathcal{V}_i(\cdot, x_{-i}) : \mathbb{R}_+^{\ell+k} \to \mathbb{R}$ is locally non-satiated and quasi-concave.

(A.2) Each component f_j of the function f is a continuous, non-decreasing and convex function. In addition, f_j is strictly increasing in the consumption of commodity $\ell+j$. Moreover, $f(\hat{z}, 0) = 0$, being $(\hat{z}, 0)$ any bundle with no consumption of the commodities that originate externalities.

Remark 1. Consider a command and control regulation which sets a numerical quantity limit L_j to the use of each commodity $\ell + j$ that has external effects. Note that if for every $j = 1, \dots k$, we define f_j as the consumption of commodity $\ell + j$, i.e., $f_j(z) = z^{\ell+j}$, and the total amount of rights is such that $L = R$, then any feasible allocation x fulfills the cap.

Let $p = (p_1, \dots, p_{\ell+k})$ denote a price vector for commodities and $q \in \mathbb{R}_+^k$ a price vector for rights. For every $(p, q) \in \mathbb{R}_+^{\ell+k}$ the budget set of the consumer i is

$$B_i(p, q) = \left\{x \in \mathbb{R}_+^{\ell+k} | p \cdot x + q \cdot f(x) \leq p \cdot \omega_i + q \cdot r_i\right\}$$

An equilibrium is a price vector (p, q) for commodities and rights and a feasible allocation $x = (x_i, i \in N)$, such that the bundle x_i maximizes $\mathcal{V}_i(\cdot, x_{-i})$ on the budget set $B_i(p, q)$ for every i.

Remark 2. Since preferences are locally non-satiated, at equilibrium we have

$$p \cdot \sum_{i\in N} (x_i - \omega_i) + q \cdot \sum_{i\in N} (f(x_i) - r_i) = 0.$$

Then, taking into account the feasibility conditions, if $\sum_{i\in N} f_j(x_i) < R_j$ we have $q_j = 0$ and, in addition, $p_{\ell+j} = 0$ whenever $\sum_{i\in N} x_i^{\ell+j} < \sum_{i\in N} \omega_i^{\ell+j}$. Moreover, if $\mathcal{V}_i$ is increasing in the consumption of the commodity $\ell + j$, we conclude that at any equilibrium where there is an effective cap on the consumption of the commodity $\ell + j$ (i.e. $\sum_{i\in N} x_i^{\ell+j} < \sum_{i\in N} \omega_i^{\ell+j}$), then the price of this commodity becomes null and the relevant price is the price of the rights.

Applying the results in Hervés-Beloso, Martinez and Rivera (2012) we can obtain existence of equilibrium for our economy.

A non-efficiency example. Consider an economy with two consumers (1 and 2) and two commodities (x and y). In order to consume the good y rights are necessary and are given by $f(y) = y$. Each agent is endowed with 2 units of every commodity. Agent 1 has 2 units of rights whereas agent 2 has 1 unit of rights.

As in the previous example, the preferences are represented by the following utility functions:

$$\mathcal{V}_1\left((x_1, y_1), (x_2, y_2)\right) = \begin{cases} x_1 y_1 \text{ if } y_1 + y_2 \leq 2.5 \\ x_1 y_1 - (y_1 + y_2 - 2.5) \text{ otherwise} \end{cases}$$

$$\mathcal{V}_2\left((x_1, y_1), (x_2, y_2)\right) = x_2 y_2.$$

Note that any allocation which distributes the total endowment of y is not feasible.

A Walrasian equilibrium is given by the consumption bundles $(5/2, 3/2)$ for consumer 1 and $(3/2, 3/2)$ for consumer 2, commodity prices $p_x = 1, p_y = 0$ and right price $q = 1$.

However this equilibrium is not efficient. To see this, note that the allocation which assigns to consumer 1 the consumption bundle $\left(\left(\frac{5}{2} - a\right), \left(\frac{3}{2} + b\right)\right)$ and to consumer 2 the bundle $\left(\left(\frac{3}{2} + a\right), \left(\frac{3}{2} + b\right)\right)$ is feasible. With this allocation every individual is better off whenever $b \in (0, 3/2)$ and a belongs to the interval $\left(\frac{3b}{3-2b}, \frac{5b}{3+2b}\right)$. Take, for instance, $a = 1/3$ and $b = 1/4$ and note that

$$\mathcal{V}_1\left(\left(\tfrac{13}{6}, \tfrac{7}{4}\right), \left(\tfrac{11}{6}, \tfrac{5}{4}\right)\right) = \tfrac{13}{6} \cdot \tfrac{7}{4} - (3 - 2.5) > \tfrac{5}{2} \cdot \tfrac{3}{2} - (3 - 2.5) = \mathcal{V}_1\left(\left(\tfrac{5}{2}, \tfrac{3}{2}\right), \left(\tfrac{3}{2}, \tfrac{3}{2}\right)\right)$$

$$\mathcal{V}_2\left(\left(\tfrac{13}{6}, \tfrac{7}{4}\right), \left(\tfrac{11}{6}, \tfrac{5}{4}\right)\right) = \tfrac{11}{6} \cdot \tfrac{5}{4} > \tfrac{3}{2} \cdot \tfrac{3}{2} = \mathcal{V}_2\left(\left(\tfrac{5}{2}, \tfrac{3}{2}\right), \left(\tfrac{3}{2}, \tfrac{3}{2}\right)\right)$$

We remark that this inefficiency at equilibrium is basically due to the fact that the aggregate equilibrium consumption of the second commodity is above the cap from which the externality appears and at the same time it is lower than the total endowment of such a commodity since the total amount of rights does not allow market clearing of this commodity.

Then, within this general equilibrium framework, the assignment of rights and the presence of a positive level below which the externality is negligible are not enough to get efficiency. Therefore, additional assumptions are required to get a version of the first welfare theorem for this setting.

3 Revisiting the Coase theorem

We remind that the Coase theorem, as named and formulated by Stigler (1966), points out that complete property rights and zero (or low) transaction costs are needed to get an efficient market solution.

Now our aim is to revisit the Coase theorem in a general equilibrium scenario with a "*cap and trade system*", in order to get an efficient market solution. Moreover, in which follows we obtain a version of the first welfare theorem in its strong version. That is, we show that the equilibrium belongs to the core and, in particular, is efficient.

Attempting to get a version of the first welfare theorem for our model, we face a conceptual problem which is the definition of the core. Given that each individual preference depends on others' consumption choices, how should we evaluate the actions of agents outside of a coalition once the coalition forms? This problem does not arise in the classical case when agent i's preferences depend only on her consumption, but it raises important issues in our context. In fact, for economic environments with externalities, there can be many definitions of the core. This is because after a deviation, the payoff of the deviating group depends on what the complementary coalition does. Thus, one has to make assumptions about what a deviating coalition conjectures about the reaction of the others while defining the core.

To define the core, let S be a coalition; given an allocation x we write $x = (x_S, x_{-S})$, where x_S denotes the consumption bundle assigned to members of S whereas x_{-S} are the consumption bundles assigned to agents that do not belong to S, i.e., to agents in $N \setminus S$.

An allocation $x_S = (x_i, i \in S)$ is attainable for S if

(i) $\sum_{i \in S} x_i \leq \sum_{i \in S} \omega_i$ (*physical feasibility for* S) and

(ii) $\sum_{i \in S} f(x_i) \leq \sum_{i \in S} r_i$ (*rights feasibility for* S).

Several different blocking notions have been proposed in the literature. In the work by Makarov and Vasil'ev (1984) it is consider that S blocks x if there is $y_S = (y_i, i \in S)$ attainable for S such that each $i \in S$ prefers $(y_S, 0)$ rather than x. Florenzano (1989, 1990), addressing non-ordered preferences, sets that S blocks x if there is $y_S = (y_i, i \in S)$ attainable for S such that each $i \in S$ prefers (y_i, x_{-i}) rather than x. More recently, Dufwenberg *et al.* (2010): consider the following; S blocks x if there is y_S attainable for S such that every $i \in S$ prefers (y_S, x_{-S}) rather than x.

Note that the the first notion assumes that agents outside the coalition get a null bundle and the last two notions deal with allocations that may not be feasible.

To our purposes, when a coalition blocks an allocation we need to consider not only the consumption bundles of every agent in the coalition but also what the coalition expect about the actions of the others.

Next, we state different specifications of the veto system that result in different core solutions. These definitions are inspired by the work of Aumann (1964) on the core of a cooperative game without side payments.

Pessimistic Core.

Consider that coalitions expect that the behavior of outsiders leads to the worst situation for them That is, coalitions are pessimistic about what they can achieve. More precisely, an allocation x is strongly blocked by the coalition S if there exists an attainable allocation y_S for S such that

$$\mathcal{V}_i(y_S, y_{-S}) > \mathcal{V}_i(x), \quad \text{for every } i \in S,$$

for every attainable allocation y_{-S} for $N \setminus S$. That is, what the coalition can guarantee itself improves its members regardless of the actions of outsiders.

Optimistic Core.

Coalitions may also be optimistic and adopt a positive thinker behavior. If there is a possibility of improving they will be able to become better off. Thus, an allocation x is weakly blocked by the coalition S if there exists an attainable allocation y_S for S such that

$$\mathcal{V}_i(y_S, y_{-S}) > \mathcal{V}_i(x), \quad \text{for every } i \in S,$$

for some attainable allocation y_{-S} for $N \setminus S$.

Cautious Core.

We can also consider cooperative solutions in-between. For instance, coalitions may be cautious and adopt foresight behavior. Then, an allocation x is prudent blocked by the coalition S if for each attainable allocation y_{-S} for $N \setminus S$, there exists an attainable allocation y_S for S such that

$$\mathcal{V}_i(y_S, y_{-S}) > \mathcal{V}_i(x), \quad \text{for every } i \in S.$$

We remark that the harder to block an allocation is, the larger the core we obtain. Then, we have

$$\text{Optimistic core} \subseteq \text{Cautious core} \subseteq \text{Pesimistic core}.$$

A feasible allocation is efficient if it is not blocked by the coalition formed by all the agents in the economy. That is, a feasible allocation x in the economy $\mathcal{E}$ is efficient if there is no feasible allocation y such that $\mathcal{V}_i(y) > \mathcal{V}_i(x)$ for every $i \in N$. We remark that the set of efficient allocations depends on preferences and on the total endowments of commodities and rights but is independent of the initial distributions of both goods and rights. Note that when the blocking coalition is the big coalition, then the strong, weak and prudent veto are the same. We remark that the set of efficient allocations depends on preferences and on the total endowments of commodities and rights but is independent of the initial distributions of both goods and rights.

To obtain a version of the first welfare theorem within our framework, we state the following assumption:

(A.3) Assume that f and the total rights R are defined in such a way that $\sum_{i \in N} f(x_i) \leq R$ implies that $\sum_{i \in N} \tilde{x}_i < L$.

Let $\hat{L} \ll L$. Consider that for every $j = 1, \ldots, k$, the mapping f_j depends only on the consumption of commodity $\ell + j$, that is, $f_j(z) = g_j(z^{\ell+j})$, and in addition g_j is any convex function such that $n g_j\left(\frac{\hat{L}_j}{n}\right) = R_j$. Then, for any feasible allocation x we have $\sum_{i \in N} \tilde{x}_i \leq \hat{L}$ and therefore (A.3) holds. To show this, note that if $\sum_{i \in N} \tilde{x}_i^h > \hat{L}_h$ for some $h \in \{1, \ldots, k\}$, then by convexity and strict monotonicity $g_h\left(\sum_{i \in N} \tilde{x}_i^h\right) \geq n g_h\left(\frac{\sum_{i \in N} \tilde{x}_i^h}{n}\right) > n g_h\left(\frac{\hat{L}_h}{n}\right) = R_h$, in contradiction with the feasibility condition.

Theorem 3.1 *Under assumptions (A.1), (A.2) and (A.3) we have the following statements:*

(i) The set of equilibria in the economy $\mathcal{E}$ with externalities coincides with the set of equilibria in the economy $\hat{\mathcal{E}}$ without externalities where preferences of each agent i are given by U_i instead of $\mathcal{V}_i$.

(ii) An equilibrium in the economy with externalities belongs to the optimistic core that, in this case, coincides with the pessimistic core. In particular, the equilibrium is efficient.

Proof. To show *(i)* let (p, q, x) be an equilibrium in $\mathcal{E}$ and assume that it is not equilibrium in $\hat{\mathcal{E}}$. Then there is an agent h and a bundle z such that $z \in B_h(p, q)$ and $U_i(z) > U_i(x_i)$. If $\sum_{i \neq h} \tilde{x}_i + \tilde{z} \leq L$ we obtain a contradiction. Otherwise, $z_\lambda = \lambda z + (1 - \lambda)x_h \in\in B_h(p, q)$ for every $\lambda \in [0, 1]$ and $\sum_{i \neq h} \tilde{x}_i + \tilde{z}_\lambda \leq L$ for all $\lambda \leq \hat{\lambda}$ which implies $\mathcal{V}_h(z_\lambda, x_{-h}) = U_h(z_\lambda)$. By convexity of preferences $U_h(z_\lambda) > U_h(x_h) = \mathcal{V}_h(x)$ which is a contradiction.

To show the converse, assume let (p, q, x) be an equilibrium in $\hat{\mathcal{E}}$ that it is not equilibrium in $\mathcal{E}$. Then there is an agent h and a bundle z such that $z \in B_h(p, q)$ and $\mathcal{V}_h(z, x_{-h}) > \mathcal{V}_h(x_h, x_{-h}) = U_h(x_h)$. Taking z_λ as before and following the same argument we get a contradiction.

To show *(ii)*, assume that (p, q, x) be an equilibrium in $\mathcal{E}$ and x is not in $C(\mathcal{E})$. Then, there is y_S attainable for a coalition S such that for some feasible allocation $y = (y_S, y_{-S})$ we have $\mathcal{V}_i(y) > \mathcal{V}_i(x)$, for every $i \in S$. Assumption (A.3) guarantees that $\mathcal{V}_i(y) = U_i(y_i)$ and $\mathcal{V}_i(x) = U_i(x_i)$ for every i. Then for every $i \in S$ we have that y_i does not belong to $B_i(p, q)$ and in this way we get a contradiction with the feasibility of y_S for the coalition S.

Finally, we remark that assumption (A.3) implies that the weak and strong core coincide provided that for every feasible allocation x we have $\mathcal{V}_i(x) = U_i(x_i)$ for every $i \in N$.

Q.E.D.

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