Bargaining set with endogenous leaders: A convergence result

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Abstract. We provide a notion of bargaining set with endogenous proponents for finite economies, and show its convergence to the Walrasian allocations when the economy is replicated.

JEL Classification: D51, D11, D00.

Keywords: Bargaining sets, leader, coalitions, core, veto mechanism.

* This work is partially supported by Research Grant SA072U16 (Junta de Castilla y León).

1 Introduction

The core of an economy is defined as the set of allocations which cannot be blocked or objected by any coalition. Thus, the veto mechanism that defines the core does not take into account that other agents in the economy may react to an objection and propose an alternative or counterobjection.

This two-step conception of the veto mechanism was considered by Aumann and Maschler (1964), who introduced the concept of bargaining set, containing the core of a cooperative game.[1] In the Aumann and Maschler and Davis and Maschler (1963) definitions, the original objection is proposed by a "leader" that must be excluded in any counterobjecting coalition.

Geanakoplos (1978) considered sequences of transferable utility (TU) exchange economies with smooth preferences and modifieded the Aumann-Davis-Maschler denition so that the "leader" was a group of agents containing a fixed (but small) fraction of the number of agents in the economy; thus, as the number of agents grew along the sequence of economies, the number of individuals in the "leader" grew proportionately. By using nonstandard analysis, he showed that this Geanakoplos bargaining set becomes asymptotically competitive as the number of agents grows. Shapley and Shubik (1984) showed that the Aumann-Davis-Maschler bargaining set is approximately competitive in replica sequences of TU exchange economies with smooth preferences. Anderson (1998) extended both Geanakoplos' result to nontransferable utility (NTU) exchange economies without smooth preferences and the Shapley and Shubik result to non-replica sequences of NTU exchange economies with smooth preferences.

On the other hand, Mas-Colell (1989) considered (NTU) economies with a continuum of agents and proposed a modification of the Aumann-Maschler bargaining set that does not involve the concept of a leader. Under conditions of generality similar to the Aumann's (1964) core equivalence theorem, he showed that his bargaining set and the set of Walrasian allocations coincide. Anderson, Trockel and Zhou (1997) showed the Mas-Colell (1989) and Zhou (1994) bargaining sets need not converge in replica sequences of economies, no matter how nice the preferences may be.

The presence of a "leader" that proposes the objection and precommits not to participate in any counterobjection makes it easier to create a justified objection.

[1]Maschler (1976) discussed the advantages that the bargaining set has over the core.

Indeed, it is remarkable that the designation of a leader makes a profound difference at a conceptual level in the resulting bargaining set and specially regarding convergence properties.

In this paper, we provide a notion of bargaining set where the potential proponents of the objections are determined endogenously and are understood as types of agents that are fully represented in the corresponding coalitions. This implies that our solution differs from the previous one considered in the related literature. Moreover, the notion of endogenous leader in the objection process allows us show that the corresponding bargaining set shrinks and converges to the set of Walrasian allocations when the economy is replicated.

Our approach is in the spirit of the work by Debreu and Scarf (1963). To be precise, the notion of bargaining set with endogenous leader we provide is based on the one that Hervés-Estévez and Moreno-García (2016) define without the consideration of leaders. In this companion paper it is obtained a convergence theorem under a necessary assumption of continuity of the Walrasian equilibrium correspondence, which is a limitation. We show that the presence of endogenous leaders leads us to a limit result with no continuity condition on the equilibrium correspondence.

The rest of the work is structured as follows. In Section 2, we collect notations and preliminaries. In Section 3, we state the notion of justified objections with endogenous leaders that is considered to define the leader bargaining set. In Section 4, we obtain a limit result for the bargaining set with endogenous leaders we provide. Finally, Section 5 contains some concluding remarks.

2 Preliminaries and notations

Let $\mathcal{E}$ be an exchange economy with a finite set of agents $N = \{1, \ldots, n\}$, who trade a finite number ℓ of commodities. Each consumer i has a preference relation $\succsim_i$ on the set of consumption bundles $\mathbb{R}^\ell_+$, with the properties of continuity, convexity[2] and strict monotonicity. This implies that preferences are represented by utility functions $U_i, i \in N$. Let $\omega_i \in \mathbb{R}^\ell_{++}$ denote the endowments of consumer

[2]The convexity of preferences we require is the following: If a consumption bundle z is strictly preferred to $\hat{z}$ so is the convex combination $\lambda z + (1 - \lambda)\hat{z}$ for any $\lambda \in (0, 1)$. This convexity property is weaker than strict convexity and it holds, for instance, when the utility functions are concave.

i. So the economy is $\mathcal{E} = (\mathbb{R}^\ell_+, \succsim_i, \omega_i, i \in N)$.

An allocation x is a consumption bundle $x_i \in \mathbb{R}^\ell_+$ for each agent $i \in N$. The allocation x is feasible in the economy $\mathcal{E}$ if $\sum_{i=1}^n x_i \leq \sum_{i=1}^n \omega_i$. A price system is an element of the $(\ell - 1)$-dimensional simplex of $\mathbb{R}^\ell_+$. A Walrasian equilibrium for the economy $\mathcal{E}$ is a pair (p, x), where p is a price system and x is a feasible allocation such that, for every agent i, the bundle x_i maximizes the utility function U_i in the budget set $B_i(p) = \{y \in \mathbb{R}^\ell_+ \text{ such that } p \cdot y \leq p \cdot \omega_i\}$. We denote by $W(\mathcal{E})$ the set of Walrasian allocations for the economy $\mathcal{E}$.

A coalition is a non-empty set of consumers. An allocation y is said to be attainable or feasible for the coalition S if $\sum_{i\in S} y_i \leq \sum_{i \in S} \omega_i$. Let $x \in \mathbb{R}^{\ell n}_+$ be a feasible allocation in the economy $\mathcal{E}$. The coalition S blocks x if there exists an allocation y which is attainable for S, such that $y_i \succsim_i x_i$ for every $i \in S$ and $y_j \succ_j x_j$ for some member j in S. When S blocks x via y we say that (S, y) is an objection to x. A feasible allocation is efficient if it is not blocked by the grand coalition, formed by all the agents. The core of the economy $\mathcal{E}$, denoted by $C(\mathcal{E})$, is the set of feasible allocations which are not blocked or objected by any coalition of agents.

It is known that, under the hypotheses above, the economy $\mathcal{E}$ has Walrasian equilibrium and that any Walrasian allocation belongs to the core (in particular, it is efficient).

Along this paper, we will refer to sequences of replicated economies. For each positive integer r, the r-fold replica economy $r\mathcal{E}$ of $\mathcal{E}$ is a new economy with rn agents indexed by ij, $j = 1, \dots, r$, such that each consumer ij has a preference relation $\succsim_{ij} = \succsim_i$ and endowments $\omega_{ij} = \omega_i$. That is, $r\mathcal{E}$ is a pure exchange economy with r agents of type i for every $i \in N$. Given a feasible allocation x in $\mathcal{E}$ let rx denote the corresponding equal treatment allocation in $r\mathcal{E}$, which is given by $rx_{ij} = x_i$ for every $j \in \{1, \dots, r\}$ and $i \in N$.

In addition, we will use the fact that, regarding Walrasian equilibria, a finite economy $\mathcal{E}$ with n consumers can be associated to a continuum economy $\mathcal{E}_c$ with n-types of agents as we specify next. Given the finite economy $\mathcal{E}$, let $\mathcal{E}_c$ be the associated continuum economy, where the set of agents is $I = [0,1] = \bigcup_{i=1}^n I_i$, with $I_i = \left[\frac{i-1}{n}, \frac{i}{n}\right)$ if $i \neq n$; $I_n = \left[\frac{n-1}{n}, 1\right]$; and all the agents in the subinterval I_i are of the same type i. That is, every agent $t \in I_i$ has preferences $\succsim_t = \succsim_i$ and endowments $\omega(t) = \omega_i$. In this case, $x = (x_1, \dots, x_n)$ is a Walrasian allocation in $\mathcal{E}$ if and only if the step function f_x (defined by $f_x(t) = x_i$ for every $t \in I_i$)

is a competitive allocation in $\mathcal{E}_c$.

Mas-Colell (1989) provided a notion of bargaining set for continuum economies, that we denote by B_{MC}, and show its coincidence with the competitive allocations. The Mas-Colell's bargaining set contains all the feasible allocations of the economy which, if objected, they could also be counterobjected, being the definition of counterobjection as follows:

Let (S, y) be an objection to the allocation f in the atomless economy $\mathcal{E}_c$. A counterobjection to (S, y) is a pair (T, z), where z is an attainable allocation for the coalition T, such that $z(t) \succ_t y(t)$ for almost every $t \in T \cap S$ and $z(t) \succ_t f(t)$ for almost every $t \in T \setminus S$.

Note that no relation is required between the coalition that objects an allocation and the coalition that counterobjects.

3 Justified objections with endogenous leaders

The concept of bargaining set depends on how justified objections are stated. Indeed, since the original definition by Aumann and Maschler (1964), a variety of different notions have been subsequently proposed. In this paper, we build upon the concept of justified* objection provided by Hervés-Estévez and Moreno-García (2016) and define a notion of bargaining set which involves the concept of endogenous leader as potential proponents that support an objection.

For it, let x be a feasible allocation in the economy $\mathcal{E}$. Consider that the coalition S blocks x via y. The objection (S, y) is counterobjected in the replicated economy $r\mathcal{E}$ if there exist a set of types $\mathcal{T} \subset N$, an equal treatment allocation $(z_i, i \in \mathcal{T})$ and natural numbers $n_i \leq r, i \in \mathcal{T}$, such that

(i) $\sum_{i \in \mathcal{T}} n_i z_i \leq \sum_{i \in \mathcal{T}} n_i \omega_i$ and

(ii) $z_i \succ_i y_i$ for every $i \in \mathcal{T} \cap S$ and $z_i \succ_i x_i$ for every $i \in \mathcal{T} \setminus S$.

An objection is justified* if it is not counterobjected in any replicated economy. A feasible allocation belongs to $B^*(\mathcal{E})$ if it has no justified* objection.Thus, this notion relies crucially on the set of types of agents that participate in the coalitions that object or counterobject, respectively. Under continuity of the Walrasian equilibrium correspondence, Hervés-Estévez and Moreno-García (2016) show that, as it occurs with the core, the bargainng set $B^*(r\mathcal{E})$ shrinks to the Walrasian allocations when r increases. This is in contrast to the non-convergence result by Anderson, Trockel and Zhou (1997) who adapt Mas-Colell's bargaining set to finite economies in a different way showing that it does not shrink to the set of Walrasian allocations in a sequence of replicated economies.

Neither the concept of bargaining set by Mas-Colell (1989) nor the notion of justified* objection imposes any restriction on the members that may belong to an objecting or counterobjecting coalition. However, this is not the case for the original definition of bargaining set introduced by Aumann and Maschler (1964) and Davis and Maschler (1963), neither for most of the papers that obtain convergence results for bargaining sets (see Geanakoplos, 1978, Shapley and Shubik, 1984 and Anderson, 1998). In these works, it is required either the presence of a leader or a group of leaders who acts as proponent of the original objection. The underlying argument is that when an objection is proposed by a leader, any counterobjecting coalition must exclude this leader.

In which follows, we provide a notion of bargaining set with endogenous leaders as proposer and defender of an objection. For it, consider an equal-treatment objection (S, y) to the allocation rx in $r\mathcal{E}$. That is, there are $r_i \leq r$ agents of each type $i \in S$ such that $\sum_{i\in S} r_i y_i \leq \sum_{i\in S} r_i \omega_i$ and $y_i \succsim_i x_i$ for every $i \in S$, with strict preference for some $j \in S$. We remark that without loss of generality we assume $r_h = r$ for some $h \in S$.[3] Let $\mathcal{L}_S = \{i \in S | r_i = r\}$. We say that any type in $\mathcal{L}_S$ can behave as a proponent of the objection (S, y). That is, any type that is fully represented in the blocking system may become a proposer who takes the role of a leader that supports the corresponding objection. Thus, we consider that the proposers of an objecting allocation depend on the own membership of the involved coalitions and therefore the potential leaders are endogenously determined. Note that when we refer to the original economy $\mathcal{E}$, i.e. $r = 1$, for any objection (S, y) we have $\mathcal{L}_S = S$ and every member in the coalition may behave as a leader. To be precise, we state the following definitions.

Definition 3.1 *An objection (S, y) to rx in the economy $r\mathcal{E}$ is $\mathcal{L}$-justified if it is equal-treatment and there exists $i \in S$, with $r_i = r$, such that any counterobjection (T, z) in $\hat{r}\mathcal{E}$ with $\hat{r} \geq r$ requires that i belongs to T. In other words, the equal-tratment objection (S, y) to rx in the economy $r\mathcal{E}$ is $\mathcal{L}$-counterobjected if for every $i \in S$, with $r_i = r$, there exists a counterobjection (T, z), with $i \notin T$, in some replicated economy $\hat{r}\mathcal{E}$ with $\hat{r} \geq r$.*

[3] Note that otherwise we can consider the objection (S, y) in the replicated economy $\bar{r}\mathcal{E}$ with $\bar{r} = \max_{i\in S} r_i$.

Definition 3.2 *We say that the feasible allocation x belongs to the leader bargaining set of $r\mathcal{E}$ and we write $x \in B_{\mathcal{L}}(r\mathcal{E})$ if the allocation rx has no $\mathcal{L}$-justified objection.*

We stress that in our definition, a potential leader, besides being endogenously determined, consists in a group formed by all the individuals of the same type and every type that participates with all its agents in an objection can be designated as a leader. Consequently, in our notion a leader becomes a type and thus the measure of any leader is maintained when the economy is replicated and is given by 1/n. Therefore, the concept of leader justified objection that we provide and, in turn, our leader bargaining set differ from those that have already been stated in the previous related literature.

We must remember that, for every r, the set of Walrasian allocations of the economy $\mathcal{E}$ is contained in the core of $r\mathcal{E}$ which is contained in $B_{\mathcal{L}}(r\mathcal{E})$. Moreover, according to our leader bargaining set, for any natural number r, there is $\hat{r} \geq r$ such that $B_{\mathcal{L}}(\hat{r}\mathcal{E}) \subseteq B_{\mathcal{L}}(r\mathcal{E})$. To see this, note that obviously we have $B_{\mathcal{L}}(2r\mathcal{E}) \subseteq B_{\mathcal{L}}(r\mathcal{E})$.

4 A convergence result

It is important to remark that the designation of a leader in the objecting mechanism makes a profound difference in the resulting bargaining sets, especially when the economy is enlarged with the aim of studying convergence properties. Indeed, most of the bargaining sets convergence results that have been obtained depend crucially on the presence of a leader or a group of leaders (see Geanakoplos, 1978, Shapley and Shubik, 1984 and Anderson, 1998). Roughly speaking, the aforementioned asymptotic results show that different notions of bargaining set involving the presence of a leader can approximately be decentralized by prices for large economies.

The notion we have provided in the previous section, specifies endogenous leaders as proposers of an objection and allows us to show that when we replicate the economy, the resulting bargaining set converges to the set of Walrasian allocations, in a similar way as the Debreu-Scarf's limit theorem for the core, without any additional continuity property of the equilibrium correspondence as it has been required for the aforementioned convergence result by Hervés-

Estevez and Moreno-García (2016). Our convergence result depends crucially on the consideration of endogenous leaders understood as types.

Theorem 4.1 *The allocation x is Walrasian in the economy $\mathcal{E}$ if and only if x belongs to the leader bargaining set of every replicated economy. That is,*

$$\bigcap_{r\in\mathbb{N}} B_{\mathcal{L}}(r\mathcal{E}) = W(\mathcal{E}).$$

Proof. Since $W(\mathcal{E}) \subset C(r\mathcal{E}) \subset B(r\mathcal{E})$, it is immediate that $W(\mathcal{E}) \subseteq \bigcap_{r\in\mathbb{N}} B_{\mathcal{L}}(r\mathcal{E})$.

To show the converse, consider $x \in \bigcap_{r\in\mathbb{N}} B_{\mathcal{L}}(r\mathcal{E})$ and assume that x is not a Walrasian allocation in the economy $\mathcal{E}$. Let us consider the corresponding step function f_x in the associated continuum economy $\mathcal{E}_c$. We have that f_x does not belong to $B_{MC}(\mathcal{E}_c)$. Then, there exists a justified objection to f_x following Mas-Colell's definition in $\mathcal{E}_c$. By convexity of preferences, Remark 5 in Mas-Colell (1989) allows us to ensure that there is a justified objection to x that is given by (S, y) and parameters $\alpha_i, i \in S$, such that $\sum_{i\in S} \alpha_i y_i \leq \sum_{i\in S} \alpha_i \omega_i,\ y_i \succsim_i x_i$ for every $i \in S$ and $y_j \succ_j x_j$ for some $j \in S$. Moreover, $\alpha_j = 1$ and $y_i \sim_i x_i$ for every i such that $\alpha_i < 1$.

If $S = \{j\}$ the pair $(\{j\}, y_j)$ is an objection in every replicated economy. Then, for every $r\mathcal{E}$ there is a collection T of types which excludes j and an allocation z such that (T, z) counterobjects $(\{j\}, y_j)$. Then we can find a counterobjection in $\mathcal{E}_c$ to the justified objection, which is a contradiction.

Now consider that S contains not only the type j. By continuity of preferences, we can take ε such that $(1-\varepsilon)y_j \succ_j x_j$. Let $\alpha = \sum_{\substack{i\in S\\ i\neq j}} \alpha_i$ and define the allocation $\tilde{y}$ as follows:

$$\tilde{y}_i = \begin{cases} (1-\varepsilon)y_i & \text{if } i = j \\ y_i + \frac{\varepsilon y_j}{\alpha} & \text{if } i \neq j \end{cases}$$

By construction, $\sum_{i\in S} \alpha_i \tilde{y}_i \leq \sum_{i\in S} \alpha_i \omega_i$. Since preferences are monotone $\tilde{y}_i \succ_i x_i$ for every $i \in S$. Actually, $\tilde{y}_i \succ_i y_i \succsim_i x_i$, for every $i \neq j$.

For every natural $k \in \mathbb{N}$, let α_i^k, $i \in S$ be the smallest integer greater than or equal to $k\alpha_i$. Let us denote $y_i^k = \frac{k\alpha_i}{\alpha_i^k}(\tilde{y}_i - \omega_i) + \omega_i$. Note that y_i^k converges to $\tilde{y}_i$ for every $i \in S$ and then, by continuity of preferences, we have that $y_i^k \succ_i x_i$ for every $i \in S$ and for all k large enough. In addition, $y_i^k \succ_i y_i \succsim_i x_i$ for every $i \neq j$ and for all k large enough. We remark that $y_j^k = (1-\varepsilon)y_j$ and $\alpha_j^k = 1$ for every k.

Then, the coalition with α_i^k agents of type $i \neq j$ with $i \in S$, and k agents of type j, blocks x via y^k in the replicated economy $k\mathcal{E}$. Therefore, there exists a counterobjection (T, z) to the objection (S, y^k) in some replicated economy $r\mathcal{E}$ with $r \geq k$, such that $j \notin T$. Thus, for every $i \in T$, there exists a natural number $\beta_i \leq r$, such that $\sum_{i \in T} \beta_i z_i \leq \sum_{i \in T} \beta_i \omega_i$, $z_i \succ_i y_i^k \succ_i y_i$ for every $i \in T \cap S$ and $z_i \succ_i x_i$ for every $i \in T \setminus S$. This is a contradiction with the fact that the objection (S, y) defines a justified objection to f_x in the associated continuum economy.

Q.E.D.

5 Final Remarks

Our convergence theorem adds to the line of research showing that it makes a fundamental difference for the asymptotic analysis of the bargaining sets whether one requires that there be a group of leaders or not.

The notion of the bargaining set with leader we state differs from those which have been considered in the related literature and, in turn, neither our convergence result can be deduced from the previous ones nor vice-versa. Moreover, we show that the intersection of the bargaining sets of the sequence of the replicated economies coincides with the set of Walrasian allocations, providing an extension of the Debreu-Scarf core-convergence to bargaining sets which is not the case of the already obtained asymptotic theorems that show a convergence in measure (Anderson, 1998). We also remark that Anderson's convergence result applies to quite general sequences of finite exchange economies while ours is restricted to replica sequences of economies.

Finally, we point out that the consideration of endogenous leaders in the objecting system becomes the main reason that allows us to drop the restrictive assumption on continuity of the equilibrium correspondence that is a necessary requirement for the limit result on bargaining sets recently obtained by Hervés-Estévez and Moreno-García (2016) where there is no prerequisite regarding the presence of leaders in the coalitions that propose an objection.

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