Working Paper Series

Congestion-reducing investments and tying as product differentiation with price regulation

José María Chamorro Rivas
3-08
Congestion-reducing investments and tying as product differentiation with price regulation*

Jose-Maria Chamorro-Rivas†
Department of Applied Economics
University of Vigo
January, 2008

Abstract

In this paper, we consider two firms that produce identical products. However, the firms can sell the product with or without a service. The point is that we introduce a negative externality (a congestion in the use of the service) in the market. Finally, we also assume that firms can invest in eliminating that congestion. Our objective is to investigate the use of tying as a product differentiation strategy in a market with (a possible) congestion externality in the consumption of the tied good. We propose to combine a congestion externality model and a tying model in order to highlight the role of the congestion effects in the tying decision when there is also endogenous congestion-eliminating investments. The message of this paper is that the congestion externality may eliminate the strategic incentive to tie.

*Financial support from the Spanish Ministry of Science and Technology through DGICYT grant SEJ2005-07637-C02-01, and the Xunta de Galicia grant PGIDIT06PXIC300184PN are gratefully acknowledged.
†Facultade de Ciencias Empresariais e Turismo, Universidade de Vigo, Campus Sur, 32004 Ourense. Phone: +34 988368763. Fax: +34 988368923. E-mail: chamorro@uvigo.es.
1 Introduction

Tie-in sales require consumers to buy a product as a condition for buying another product. For example, a car dealer may offer cars with an already installed GPS, and a computer dealer may include some software packages with the sale of computer hardware. In this case, the seller ties two complementary products. However, not every instance of tying involves complementary products. For instance, a publisher may provide a magazine to a customer who purchases a newspaper. Papers on this topic have shown that if consumers have different valuations for different products, firms can increase their profits by selling the different products in one package. As an example, commodity bundling has generally been explained as a substitute for price discrimination. The gain in profit from tying is analyzed in Burstein (1960), Adams and Yellen (1976), Lewbel (1985), McAfee, et. al. (1989), and Whinston (1990).

Among other tying strategies (see, Shy, 1996), it has been shown that under oligopoly, firms may use tying tactics in order to differentiate themselves from the competing brands. Firms can increase their profit by tying their product with another product in order to differentiate itself from the competing firm; leading to market segmentation where the market splits into a group of consumers buying a basic product and another group buying the basic product tied to another good.

Following Carbajo, et al. (1990) and Horn and Shy (1996), we consider two firms that produce identical basic products. However, firms can sell this basic product with or without a service. By service, we mean medical services, telecommunication services, service repair contracts, warranties, product training, and so on. The main objective is that we introduce a negative externality (a congestion in the use of the service) in the market. Therefore, we consider that consumers achieve an increasing concern as the total level of consumption of the service rise. A casual observation shows many market economies where congestion is most common in services such as medical, transport, or telecommunications markets. For instance, the majority of medical services are more likely to be overbooked when there is a greater number of patients. In addition, the paper also assumes that firms can decide whether to invest in eliminating the congestion in the use of the service. This allows us to
recover the basic uncongested model previously analyzed in the literature.

There is a wide body of literature addressing consumption externalities for which consumers’ preferences depend on the clientele size. The reasons for this externality can be technological (see Besen and Farrell, 1994; Katz and Shapiro, 1994, for recent surveys), social, etc. A main contribution of this literature is that equilibrium prices may increase with the degree of congestion. This is because consumers’ congestion makes the demand addressed to each firm less elastic, thus reducing the incentives to lower prices. As Grilo et al. (2001) comment, this result "could explain why some people patronize distant clubs or restaurants with a small clientele".

We thus investigate the use of tying as a product differentiation strategy in a market with (a possible) congestion externality in the consumption of the tied good. For instance, some medical insurances have the option to make changes to their medical coverage by contracting dental services. In this paper, we propose to combine the congestion externality model and the tying model in order to highlight the role of the congestion effects in the tying decision when there is also endogenous congestion-eliminating investments. The message of this paper is that the congestion externality may eliminate the strategic incentive to tie. As will be shown, this leads to new results which cannot emerge in a tying model without congestion externalities. Our results show that, regardless the tying decision, no firm will undertake a congestion-eliminating investment, even with a low congestion externality or a low eliminating-investment cost. Furthermore, tying a service to differentiate products is not profitable when the consumers’ sensibility for a congested service is sufficiently high. Both firms offer the product tied with the service in order to extract the maximum consumer surplus. The reason is that, with the presence of significant congestion externalities, price competition is relaxed. Hence, the market is characterized by high equilibrium prices.

It is also found that each firm may undertake insufficient investment from a social viewpoint. The congestion-eliminating investment cost will be determinant in this result. Finally, we consider a model that incorporates price regulation. Historically, policy-makers have been inclined to regulate thoroughly some congested markets. For example, in many countries, the prices of medical services, transportation services, telecommunications, etc. have been subject to extensive regulation in terms of price. We find that governments should set a
regulated price above (below) the equilibrium price when the congestion is low (high). We also find that governments must set a regulated price below the competitive price to induce the first-best level of investment. Contrarily, literature on reciprocal externalities has found that the socially efficient price ends up being higher than the equilibrium price in order to compensate for the negative externality (see, Häckner and Nyberg, 1996).

The paper is organized as follows. Section 2 presents the model. Section 3 solves the model. Finally, in Section 4 we present the conclusions.

2 The model

We consider a duopoly where both firms (firms 1 and firm 2) produce a homogeneous product and may offer a complementary service. By service, we mean service repair contracts, warranties, product-training, and so on.

Each consumer buys at most one unit of the product, accompanied with the service if offered by the firms. We also assume that consumers are not identical in terms of preferences. They have the same valuation for the basic product. The service, however, yields different benefit to different consumers. To capture this feature of the model, let consumers be uniformly distributed with a unit density on the unit interval according to an increasing valuation for the service. A consumer indexed by $x \in [0, 1]$ derives a valuation of $s - x$ for the basic product plus the service, and a reservation value of $s - 1$ for the consumption of the basic product without the service. This specification of the consumers’ preferences implies that the consumer indexed by $x = 0$ derives the greatest valuation for the service ($s$), whereas the consumer located at $x = 1$ derives the lowest surplus ($s - 1$). Moreover, this minimum valuation for the service coincides with the valuation for the basic good without the service. We also assume that $s$ is large enough so that in equilibrium everyone buys a unit. Thus, the tied product-service is vertically differentiated from the basic product in the sense that if both are sold for the same price, each consumer prefers to have the service tied with the product.

Firms produce the basic good and offer the service at a constant marginal production cost which we assume to be zero. This is not essential to the qualitative nature of the results.
We consider there is a negative externality (a congestion in the use of the service) in the market. Congestion describes situations where consumers concern increases as the total level of consumption of the service rises. However, firms can invest in completely eliminating that congestion. We thus consider the investment decision is a dichotomic variable. Let $c_i \in \{0, c\}$ be the firm $i$’s investment decision variable with $c > 0$. If firm $i$ decides not to invest in descongestioning the service then it chooses $c_i = c$, otherwise $c_i = 0$. Thus, if firm $i$ offers the service and the investment decision is $c_i$, the firm invests $I_i (c_i) = \gamma (c - c_i)$, with $\gamma > 0$, and a consumer who buys the serviced good incurs in a congestion desutility of $c_i n_i$, where $n_i$ is the number of consumers purchasing the serviced product. The parameter $c$ represents the consumers’ sensibility for a congested service. We have assumed also that the investment increases as that consumers’ sensibility become higher.

The interaction between the firms takes place in three stages (it is a three-stage game). First, each firm decides whether to sell the product with or without a unit of the service. In the second stage, service-tying firms decide whether to invest in eliminating congestion. In the third stage, the firms compete in prices.

3 Resolution of the model

Solving for a subgame perfect equilibrium (private solution), we first characterize the third-stage price-competition equilibrium under three types of outcomes arising in the first stage of the game: (a) one firm ties the service, (b) both firms tie the service, and (c) neither does. Moreover, we compute the socially optimal provision of service and investment (social solution). A social planner chooses which firms offer the service, investment and prices. We ask, whether from a social point of view, the private solution results in a too much or too little service marketed towards consumers. We also investigate whether firms take the same investment decision in a private rather than in a social context. Finally, we consider the case where a social planner sets a regulated price and firms decide on whether to offer the service and investment. Further, we analyze whether the social planner can use the regulated price to control the investment level and the provision of the service.
3.1 One firm ties the service

Suppose that one firm sells the good tied with the service and the other without the service. Following, we will assume, without loss of generality, that firm 1 is the firm that ties the service, whereas firm 2 offers the basic product without the service.

3.1.1 Private outcome

In this section, the service-tying firm decides on investment and then, both firms set simultaneously and non-cooperatively prices.

Firstly, we derive the equilibrium prices given the investment decision. If \( p_1 \) and \( p_2 \) are the firms’ prices and \( c_1 \) is firm 1’s investment decision, a consumer indexed by \( x \in [0,1] \) is indifferent between buying the serviced and nonserviced goods when

\[
p_1 + x + c_1 n_1 = p_2 + 1. \tag{1}
\]

From the market-dividing condition (1), we have that those consumers located at the left hand side of the indifferent consumer \( x \), that is, those consumers with a higher valuation for the service than the indifferent consumer, will buy the serviced product and the rest will buy the non-serviced product. The market share of the service-tying firm is then \( n_i = x \).

Substituting \( n_i = x \) in equation (1) and working out the value of \( x \), we obtain that firms’ demands are given by \( D_1 (p_1, p_2) = \min \{ \max (x, 0), 1 \} \) and \( D_2 (p_1, p_2) = 1 - D_1 (p_1, p_2) \), where

\[
x = \frac{p_2 - p_1 + 1}{1 + c_1}.
\]

Thus, if each firm sells a positive amount, the profit functions of firms are given by

\[
\pi_1 (p_1, p_2) = p_1 \frac{p_2 - p_1 + 1}{1 + c_1} - \gamma (c - c_1), \quad \pi_2 (p_1, p_2) = p_2 \left( 1 - \frac{p_2 - p_1 + 1}{1 + c_1} \right).
\]

Maximizing these profit functions with respect to corresponding prices yields the first-order conditions (for the interior solution) given by

\[
\frac{\partial \pi_1}{\partial p_1} = \frac{p_2 - 2p_1 + 1}{1 + c_1} = 0, \quad \frac{\partial \pi_2}{\partial p_2} = \frac{p_1 - 2p_2 + c_1}{1 + c_1} = 0. \tag{2}
\]

Solving equations in (2), it follows that the Nash equilibrium prices are

\[
p_1 (c_1) = \frac{2 + c_1}{3}, \quad p_2 (c_1) = \frac{1 + 2c_1}{3}.
\]
Substituting the equilibrium prices into the profit functions, we obtain the equilibrium servicing firm’s profit function

\[ \pi_1(c_1) = \frac{(2 + c_1)^2}{9(1 + c_1)} - \gamma(c - c_1). \]  

(3)

Comparative statics show that when firm 1 invests to eliminate congestion, that is, when \( c_1 = 0 \), prices decrease. The reason for this is that the serviced product becomes more valuated by consumers and the non-servicing firm cuts the price to maintain the market share. We then observe an intense price competition. In the next proposition, we will show how this strategy will condition the investment decision of the servicing firm.

Secondly, when the Nash equilibrium prices is known, the tying-service firm chooses the level of investment \( c_1 \) that maximizes the equilibrium profit function (3).

**Proposition 1** The service-tying firm does not invest in eliminating the congestion of the service, that is, it chooses \( c_1^* = c \).

**Proof.** By comparing \( \pi_1(0) \) and \( \pi_1(c) \) in (3), it can be easily checked that \( \pi_1(0) < \pi_1(c) \), so that, firm 1 has no incentive to eliminate congestion. ■

Proposition 1 shows that the service-offering firm does not invest in eliminating the congestion of the service. The intuition behind this result is that if the servicing firm invests in eliminating the congestion of the service, the serviced product will become more attractive to consumers, reducing the nonserviced product’s market share. The nonservicing firm will react by cutting the price, intensifying the price competition. Thus, the servicing firm does not invest to avoid a price war.

The private optimal outcome is then

\[
\begin{align*}
c_1^* &= c, \\
p_1^* &= \frac{2 + c}{3}, \quad p_2^* = \frac{1 + 2c}{3}, \\
D_1^* &= \frac{2 + c}{3(1 + c)}, \quad D_2^* = \frac{1 + 2c}{3(1 + c)}, \\
\pi_1^* &= \frac{(2 + c)^2}{9(1 + c)}, \quad \pi_2^* = \frac{(1 + 2c)^2}{9(1 + c)}.
\end{align*}
\]  

(4)
3.1.2 Social outcome

In this section, a social planner takes decisions maximizing a social welfare function which is given by the consumer surplus plus firms’ profits. The social welfare function is then

\[
W(c_1, D_1) = s - \int_0^{D_1} zdz - (1 - D_1) - c_1 D_1^2 - \gamma (c - c_1).
\]  

(5)

The welfare function (5) has five terms. The first three terms collect the total sum of consumers’ valuations for the goods, the fourth term is the total congestion desutility, and the last term is the investment cost. The social planner chooses the demand \(D_1\) and investment \(c_1\) which maximize the welfare function (5).

**Proposition 2** For \(\gamma < \frac{1}{2c+1}\), the social optimal investment and demand are \(c_1^+ = 0\) and \(D_1^+ = 1\), otherwise the social optimal investment and demand are \(c_1^+ = c\) and \(D_1^+ = \frac{1}{2c+1}\).

**Proof.** Let \(c_1 = 0\), by maximizing \(W(0, D_1)\) in (5) with respect to \(D_1\), we obtain that the maximum is at \(D_1 = 1\). For \(c_1 = c\), the maximum of \(W(c, D1)\) is at \(D_1 = 1/(2c + 1)\). Therefore,

\[
W(0, 1) - W(c, 1/(1 + 2c)) = c \left( \frac{1}{2c + 1} - \gamma \right).
\]

Proposition 2 shows that for sufficiently low investment cost rate, that is, when \(\gamma < \frac{1}{2c+1}\), the social planner prefers a decongested service with all consumers purchasing the serviced product, otherwise the planner prefers a congested service for those consumers with the highest valuation for the service, while the non-service-oriented consumers will buy the basic good. Evidently, as the consumers sensibility with respect to congestion increases, that is, as the parameter \(c\) increases, the non-serviced product market’s share will also increase.

For \(\gamma < \frac{1}{2c+1}\), the social optimal outcome is then

\[
\begin{align*}
  c_1^+ &= 0, \\
  D_1^+ &= 1, \quad D_2^+ = 0, \\
  W^+ &= s - \frac{1}{2} - \gamma c,
\end{align*}
\]  

(6)
otherwise,
\[
\begin{align*}
    c_1^+ &= c, \\
    D_1^+ &= \frac{1}{2c + 1}, \\
    D_2^+ &= \frac{2c}{2c + 1}, \\
    W^+ &= s - \frac{4c + 1}{2(2c + 1)}.
\end{align*}
\] (7)

### 3.1.3 Price regulation

In this section, we consider the social planner set regulated prices to the serviced and non-serviced goods. Let \(\bar{p}_1\) and \(\bar{p}_2\) be the regulated prices of the servicing and nonservicing firms, respectively. Therefore, from the market-dividing condition (1), the indifferent consumer between the serviced and nonserviced products is at
\[
    x = \frac{\bar{p}_2 - \bar{p}_1 + 1}{1 + c_1}.
\] (8)

If each firm sells a positive amount, the servicing firm’s profit function is given by
\[
\pi_1 (c_1) = \bar{p}_1 (\bar{p}_2 - \bar{p}_1 + 1) - \frac{\bar{p}_1 (\bar{p}_2 - \bar{p}_1 + 1)}{1 + c_1} - \gamma (c - c_1).
\] (9)

The next proposition shows the optimal investment decision of the servicing firm.

**Proposition 3** For \(\gamma < \bar{\gamma} = \frac{\bar{p}_1 (\bar{p}_2 - \bar{p}_1 + 1)}{c + 1}\), the optimal investment is \(\bar{c}_1 = 0\), otherwise the optimal investment is \(\bar{c}_1 = c\).

**Proof.** Let us compare the servicing firm’s profit function (9) with and without investment. Therefore,
\[
\pi_1 (0) - \pi_1 (c) = c \left( \frac{\bar{p}_1 (\bar{p}_2 - \bar{p}_1 + 1)}{c + 1} - \gamma \right).
\]

Since the prices are regulated, the servicing firm chooses the level of investment in terms of congestion. Proposition 3 shows that the service-tying firm will invest when the investment cost is sufficiently low.

The optimal demand is then
\[
\bar{D}_1 = \frac{\bar{p}_2 - \bar{p}_1 + 1}{1 + \bar{c}_1}.
\]

Next, we determine the social optimal regulated prices \(\bar{p}_1^+\) and \(\bar{p}_2^+\). These are the prices \(\bar{p}_1\) and \(\bar{p}_2\) such that the social optimal outcome of (6) and (7) is obtained.
Proposition 4 For $\gamma < \gamma^+ = \frac{1}{2c+1}$, the social optimal regulated prices are $\bar{p}_1^+ = \bar{p}_2^+ = \frac{c+1}{2c+1}$, otherwise the social optimal regulated prices are $\bar{p}_1^+ = 1$ and $\bar{p}_2^+ = \frac{c+1}{2c+1}$.

Proof. The regulated social welfare (5) with prices $\bar{p}_1^+$ and $\bar{p}_2^+$ coincides with $W^+$ in (6) and (7) if and only if $c_1^+ = \bar{c}_1$ and $D_1^+ = \bar{D}_1$, i.e., for $\gamma^+ = \bar{\gamma}$ and $D_1^+ = \bar{D}_1$. □

Proposition 4 shows that for sufficiently low investment cost rates, that is, for $\gamma < \frac{1}{2c+1}$, regulated prices are equal to both firms, so that, all consumers buy the serviced product. However, with a high enough investment cost rate, that is, when $\gamma > \frac{1}{2c+1}$, the serviced good regulated price is higher than the non-serviced good regulated price, so that, the market is segmented, the most service-oriented consumers buy the product tied with the service, whereas, the rest of consumers do not.

For $\gamma < \frac{1}{2c+1}$, the social optimal regulated outcome is then

$$\begin{align*}
\bar{c}_1^+ &= c_1^+ = 0, \\
\bar{p}_1^+ &= \frac{c+1}{2c+1}, \quad \bar{p}_2^+ = \frac{c+1}{2c+1}, \\
\bar{D}_1^+ &= D_1^+ = 1, \quad \bar{D}_2^+ = D_2^+ = 0, \\
\bar{W}^+ &= W^+ = s - \frac{1}{2} - \gamma c, \\
\bar{\pi}_1^+ &= \frac{c+1}{2c+1} - \gamma c, \quad \bar{\pi}_2^+ = 0,
\end{align*}$$

otherwise

$$\begin{align*}
\bar{c}_1^+ &= c_1^+ = c, \\
\bar{p}_1^+ &= 1, \quad \bar{p}_2^+ = \frac{c+1}{2c+1}, \\
\bar{D}_1^+ &= D_1^+ = \frac{1}{2c+1}, \quad \bar{D}_2^+ = D_2^+ = \frac{2c}{2c+1}, \\
\bar{W}^+ &= W^+ = s - \frac{4c+1}{2(2c+1)}, \\
\bar{\pi}_1^+ &= \frac{1}{2c+1}, \quad \bar{\pi}_2^+ = \frac{2c(c+1)}{(2c+1)^2}.
\end{align*}$$

3.2 Both firms tie the service

Now, suppose that both firms tie the products with services.
3.2.1 Private outcome

If both firms decide to invest in eliminating the congestion of the service, that is, when $c_1 = c_2 = 0$, the products become homogeneous and price competition drives profits to zero. Otherwise, if $c_1$ and $c_2$ are the investment decisions, and $p_1$ and $p_2$ are the product prices, the firms’ demands are given by $D_1(p_1, p_2) = \min \{ \max (n_1, 0), 1 \}$ and $D_2(p_1, p_2) = 1 - D_1(p_1, p_2)$, where $n_1$ comes from the market-dividing condition $p_1 + c_1 n_1 = p_2 + c_2 (1 - n_1)$, that is, the market size and share of firm 1 is

$$n_1 = \frac{p_2 - p_1 + c_2}{c_1 + c_2}. \quad (12)$$

If each firm sells a positive amount, the profit functions are given by

$$\pi_1 = p_1 \frac{p_2 - p_1 + c_2}{c_1 + c_2} - \gamma (c - c_1), \quad \pi_2 = p_2 \left(1 - \frac{p_2 - p_1 + c_2}{c_1 + c_2}\right) - \gamma (c - c_1).$$

Maximizing these profit functions with respect to corresponding prices yields the first-order conditions (for the interior solution) given by

$$\frac{\partial \pi_i}{\partial p_i} = \frac{p_j - 2p_i + c_j}{c_1 + c_2} = 0.$$

Solving the first-order equations, we obtain the Nash equilibrium prices

$$p_i (c_1, c_2) = \frac{c_i + 2c_j}{3}.$$

Substituting the equilibrium prices into the profit functions, we obtain the equilibrium profits

$$\pi_i (c_1, c_2) = \frac{(c_i + 2c_j)^2}{9(c_1 + c_2)} - \gamma (c - c_1). \quad (13)$$

Secondly, knowing the Nash equilibrium prices, firms decide $c_1$ and $c_2$ that maximizes their respective equilibrium profit functions in (13).

**Proposition 5** Firms do not invest in decongesting the service, that is, $c_i^* = c$.

**Proof.** If both firms invest ($c_1 = c_2 = 0$), they obtain zero profits. Therefore, no firm will invest when the rival invests. On the contrary, from (13), we have that

$$\pi_1 (0, c) - \pi_1 (c, c) = -\frac{c(1 + 18\gamma)}{18} < 0,$$

that is, firm 1 will not invest as far as firm 2 has invested. Thus, by the symmetry of the model, we can conclude that no firm will invest as far as the rival does not invest. ■
Proposition 5 shows that if both firms tie-in the service, firms will offer a congested service, that is, \( c_1 = c_2 = c \). From (13), if neither firm invests, the firms’ profits are \( \frac{c}{2} \). However, when different decisions are taken on investment, the investing firm has profits of \( \frac{4c}{9} - \gamma c \), and the noninvesting firm has a profit of \( \frac{c}{2} \). In the last situation, the noncongested firm has four times more revenues than the congested firm. The reason for this is that price competition is strong, the congested firm has to cut the price to compensate the losses in its market share. Clearly, both firms prefer a congested service.

Price competition does not drive profits to zero when there is no investment (contrary when both firms invest) since firms’ incentives to increase their market shares are low. Note that consumers will avoid those firms with high market shares. In a model without congestion, the products become homogeneous. Therefore, price competition drives profits to zero. However, congestion exists, it makes price competition softer, allowing for positive profits.

The private optimal outcome is then

\[
\begin{align*}
    p_1^* &= c, \quad p_2^* = c, \\
    D_1^* &= \frac{1}{2}, \quad D_2^* = \frac{1}{2}, \\
    \pi_1^* &= \frac{c}{2}, \quad \pi_2^* = \frac{c}{2}.
\end{align*}
\]

### 3.2.2 Social outcome

The social planner maximizes the social welfare function

\[
W(c_1, c_2, D_1) = s - \int_0^1 zdz - c_1D_1^2 - c_2(1 - D_1)^2 - \gamma (c - c_1) - \gamma (c - c_2),
\]

with respect to \( D_1, c_1 \) and \( c_2 \).

**Proposition 6** For \( \gamma < \gamma^+ \equiv \frac{1}{2} \), the social optimal investments are \( c_1^+ = 0 \) and \( c_2^+ = c \) with \( D_1^+ = 1 \), or \( c_1^+ = c \) and \( c_2^+ = 0 \) with \( D_1^+ = 0 \), otherwise the social optimal investments are \( c_1^+ = c_2^+ = c \) with \( D_1^+ = 1/2 \).

**Proof.** For \( c_1 = c_2 = 0 \), the social welfare function is constant with respect to \( D_1 \), and \( W(0, 0, D_1) = s - \frac{1}{2} - 2\gamma c \). For \( c_1 = c_2 = c \), the maximum of the social welfare function is \( D_1 = 1/2 \) with \( W(c, c, 1/2) = s - \frac{1}{2} - \frac{\gamma}{2} \). For \( c_1 = 0 \) and \( c_2 = c \), the maximum of the social welfare function is \( D_1 = 1 \) with \( W(0, c, 1) = s - \frac{1}{2} - \gamma c \). Finally, for \( c_1 = c \) and \( c_1 = 0 \),
the maximum of the social welfare function is \( D_1 = 0 \) with \( W(c, 0, 0) = s - \frac{1}{2} - \gamma c \). Since \( W(0, 0, D_1) < W(0, c, 1) = W(c, 0, 0) \), we compare \( W(c, c, 1/2) \) and \( W(0, c, 1) \) and obtain

\[
W(c, c, 1/2) - W(0, c, 1) = c \left( \gamma - \frac{1}{2} \right).
\]

Proposition 6 shows that if the investment cost is low enough, that is, when \( \gamma < \frac{1}{2} \), the social planner prefers a decongested serviced product which is sold by one of the two firms. Otherwise, both firms share out the market and do not eliminate congestion.

For \( \gamma < \frac{1}{2} \), the social optimal welfare is then

\[
c_i^+ = 0, \quad c_j^+ = c, \quad (15)
\]

\[
D_i^+ = 1, \quad D_j^+ = 0,
\]

\[
W^+ = s - \frac{1}{2} - \gamma c,
\]

otherwise,

\[
c_i^+ = c, \quad c_j^+ = c, \quad (16)
\]

\[
D_i^+ = 1/2, \quad D_j^+ = 1/2,
\]

\[
W^+ = s - \frac{1}{2} - \frac{c}{2}.
\]

### 3.2.3 Price regulation

In this section, we consider the case in which a regulated price \( \bar{p} \) is set by the social planner. If both firms invest in decongesting the service, that is, when \( c_1 = c_2 = 0 \), the products become homogeneous and will be sold at a price \( \bar{p} \). In this case, we consider that the market is equally divided between the two firms, that is, \( D_i = 1/2 \). Otherwise, substituting \( \bar{p} \) into the market-dividing condition \( \bar{p}_1 + c_1 D_1 = \bar{p}_2 + c_2 (1 - D_1) \), we obtain that firm \( i \)'s demand is

\[
D_i = \frac{c_j}{c_i + c_j}, \quad (17)
\]
If each firm sells a positive amount, the firms’ profits are
\[
\pi_i(c_1, c_2) = \frac{\bar{p}c_i}{c_i + c_j} - \gamma (c - c_i).
\] (18)

Next, both firms set simultaneous and non-cooperative levels of investment \( c_i \).

**Proposition 7** For \( \gamma < \bar{\gamma} \triangleq \frac{\bar{p} - c}{2\gamma} \), the optimal regulated investments are \( \bar{c}_1 = \bar{c}_2 = 0 \), otherwise the optimal regulated investments are \( \bar{c}_1 = \bar{c}_2 = c \).

**Proof.** We obtain the result by comparing the following values of (18): \( \pi_1(0, 0) = \pi_2(0, 0) = \frac{\bar{p}}{2} - \gamma c \), \( \pi_1(c, c) = \pi_2(c, c) = \frac{\bar{p}}{2} \), \( \pi_1(0, c) = \pi_2(c, 0) = \bar{p} - c\gamma \), and \( \pi_1(c, 0) = \pi_2(0, c) = 0 \). ■

Substituting the optimal regulated investments into the demand functions in (17), we obtain the optimal regulated demands
\[
\bar{D}_1 = \bar{D}_2 = \frac{1}{2}.
\]

Next, we determine the social optimal regulated price \( \bar{p}^+ \). This is the price \( \bar{p} \) such that the social optimal outcome of (15) and (16) is obtained.

**Lemma 8** If \( \gamma < \frac{1}{2} \), a regulated price does not exist.

**Proof.** Both welfare values coincide if and only if \( c_i^+ = \bar{c}_i \) and \( D_i^+ = \bar{D}_i \). For \( \gamma < \frac{1}{2} \), in the social outcome (15), consumers purchase a decongested service to only one of the two firms, that is, \( D_1^+ = 1 \) and \( D_2^+ = 0 \), or \( D_1^+ = 0 \) and \( D_2^+ = 1 \). However, in the regulated price outcome, all consumers purchase a decongested service as well, but demand is equally divided between both firms, that is, \( \bar{D}_1 = \bar{D}_2 = \frac{1}{2} \). ■

For a sufficiently low investment cost rate, that is, when \( \gamma < \frac{1}{2} \), the social planner invests in decongestioning the service but gives all the sales to one of the two firms, to avoid paying twice the investment cost. When firms decide whether to invest or not, there is a lack of coordination between firms. Thus, investment costs are paid twice.

Following, we compute the second best outcome for \( \gamma < \frac{1}{2} \).

**Proposition 9** For \( \gamma < \bar{\gamma}^+ \triangleq \frac{1}{2} \), the second best optimal regulated prices and investments are \( \bar{p}^+ = \frac{\bar{c}}{2} \) and \( \bar{c}_1^+ = \bar{c}_2^+ = 0 \), respectively. Otherwise, the second best optimal regulated prices and investments are \( \bar{p}^+ = \frac{\bar{c}}{2} \) and \( \bar{c}_1^+ = \bar{c}_2^+ = c \), respectively.
Proof. From Proposition 7, firms take the same decision on investment, that is, \( \bar{c}_1 = \bar{c}_2 = 0 \) or \( \bar{c}_1 = \bar{c}_2 = c \). Comparing welfare at these two possible outcomes, we obtain that

\[
W(0, 0, 1/2) - W(c, c, 1/2) = \frac{c (1 - 4\gamma)}{2},
\]

that is, for \( \gamma < 1/4 \), the social planner prefers to eliminate congestion, otherwise it does not. Thus, let \( \bar{p}^+ \) be the regulated price such that \( \bar{\gamma} \) in Proposition 7 satisfies \( \bar{\gamma} = 1/4 \), that is, let \( \bar{p}^+ = \frac{c}{2} \).

Proposition 9 shows that if the social planner chooses the regulated price \( \bar{p} = \frac{c}{2} \), we obtain the second best solution for \( \gamma < 1/2 \), otherwise we obtain the first best outcome. The second best optimal regulated investments are then \( \bar{c}_1^+ = \bar{c}_2^+ = 0 \), when \( \gamma < 1/4 \), and \( \bar{c}_1^+ = \bar{c}_2^+ = c \), for \( 1/4 \leq \gamma \leq 1/2 \), while the first best optimal regulated investments are \( \bar{c}_1^+ = \bar{c}_2^+ = c \) when \( \gamma \geq 1/2 \).

For \( \gamma < \frac{1}{4} \), the social optimal regulated outcome is then

\[
\begin{align*}
\bar{c}_1^+ &= 0, \quad \bar{c}_2^+ = 0, \\
\bar{p}^+ &= \frac{c}{2}, \\
\bar{D}_1^+ &= \frac{1}{2}, \quad \bar{D}_2^+ = \frac{1}{2}, \\
\bar{W}^+ &= s - \frac{1}{2} - 2c, \\
\bar{\pi}_1^+ &= \frac{c}{4} - \gamma c, \quad \bar{\pi}_2^+ = \frac{c}{4} - \gamma c,
\end{align*}
\]

otherwise,

\[
\begin{align*}
\bar{c}_1^+ &= c, \quad \bar{c}_2^+ = c, \\
\bar{p}^+ &= \frac{c}{2}, \\
\bar{D}_1^+ &= \frac{1}{2}, \quad \bar{D}_2^+ = \frac{1}{2}, \\
\bar{W}^+ &= s - \frac{1}{2} - \frac{c}{2}, \\
\bar{\pi}_1^+ &= \frac{c}{4}, \quad \bar{\pi}_2^+ = \frac{c}{4}.
\end{align*}
\]
3.3 Neither firm ties services

3.3.1 Private outcome

If neither firm ties its product with the service, then the products become homogeneous, competition drives profits to zero, and the market can be arbitrarily divided between the firms.

3.4 Tying versus non-tying

Next, we compute the Nash equilibrium when firms decide whether to tie the service or not. We compare the private outcomes (4) and (14). As the results in the previous sections show, regardless the tying decision, firms will not invest in eliminating the congestion of the service. Therefore, the consumers’ sensibility for congestion \((c)\) will be the determinant in the resolution of this stage of the game.

**Proposition 10** In a three-stage game where firms choose, in the first stage, whether to tie their product with a service, and in the second and the third stages, the level of investment is choses to decongesting the service and prices, respectively, we find the following:

1. If the congestion rate is low, that is, when \(c < 1\), one firm will tie-in the service while the other will sell the product with no service.

2. If the congestion rate is high, that is, when \(c > 1\), both firms will tie-in the service.

Proposition 10 shows that we can obtain the two types of decisions and that the congestion rate \((c)\) will determine the equilibrium outcome. This result is contrasted with the one obtained in a model where congestion is not considered. In that case, only Part 1 of Proposition 10 is obtained (Shy, 1996).

It is known that under oligopoly, firms may use tying tactics in order to differentiate themselves from the competing brands. This strategy may lead to market segmentation where the market splits into a group of consumers buying the homogeneous product (the most service-oriented consumers) and another group buying a product tied to a service contract (the least service-oriented consumers). Proposition 10 shows that this is the case.
when there is practically no congestion in the use of the service, that is, when the parameter $c$ is sufficiently low. Part 1 of Proposition 10 is intuitively clear. If the consumers’ sensibility for a congested service ($c$) is sufficiently low, products become practically homogeneous. Thus, firms use tying tactics in order to differentiate themselves from the competing brands. Interestingly, in Part 2, tying a service to differentiate products is not profitable when the consumers’ sensibility for a congested service is sufficiently high. Both firms offer the product tied with the service in order to extract the maximum consumer surplus. The reason is that, with the presence of congestion externalities, rivals compete less aggressively. If the rival cuts the price, that firm does not achieve a large enough increase in the market share to offset the decreased price. This is due to consumers with high sensibility that avoid firms with larger market shares.

Note, however, that these results crucially depend on the assumption that all consumers buy the differentiated product. In a more general setting, one might expect strong congestion to lead some consumers to opt out the market.

4 The socially optimal outcome

We now ask, whether from a social point of view the private outcome equilibrium results in too much or too little service marketed to consumers, and whether private and social investment decisions coincide. According to Proposition 10, when the congestion rate is low, that is, when $c < 1$, we compare the asymmetric private outcome in (4) with the asymmetric social outcome in (6) and (7). If the congestion rate is high, that is, when $c > 1$, we compare the symmetric private outcome in (14) with the symmetric social outcome in (15) and (16). We obtain that the congestion-eliminating cost ($\gamma$) will be determinant in the comparison of these two outcomes (see Table 1).

Thus, when one firm tie-in the service and the other sells the product without service, we have the following result.

**Proposition 11** When one firm ties-in the service while the other sells the product with no service, that is, when $c < 1$, we have
1. If the eliminating-investment rate is low, that is, when \( \gamma < \frac{1}{2c+1} \), the equilibrium number of consumers purchasing the product tied with the service is lower than the socially optimal level, and the equilibrium and social optimal investment decisions do not coincide.

2. If the eliminating-investment rate is high, that is, when \( \gamma > \frac{1}{2c+1} \), the equilibrium number of consumers purchasing the product is lower than the socially optimal level, while the congestion-eliminating investment decisions coincide.

The next Proposition compares equilibrium and social investment decisions when both firms sell the product tied with the service. Evidently, in this case, all consumers purchase the product tied with the service and, thus, the serviced product’s market share is 1.

**Proposition 12** When both firms tie-in the service, that is, when \( c > 1 \), we have

1. If the eliminating-investment cost rate is low, that is, when \( \gamma < \frac{1}{2} \), the equilibrium and the social optimal congestion-eliminating investments do not coincide.

2. If the eliminating-investment cost rate is high, that is, when \( \gamma > \frac{1}{2} \), the equilibrium congestion-eliminating investment decision coincides with the socially optimal decision.

Proposition 11 shows that when the congestion rate \( (c) \) is low, having higher sales of the serviced product is socially desirable; that is, the firm that ties the product with service underproduces from a social viewpoint. The servicing firm takes an advantage of the nonservicing firm, since consumers’ valuation for the serviced product is higher than for the basic product. Thus, the nonservicing firm cuts the price trying to maintain the market share. We have also found that investment decisions do not coincide when the investment cost is low. A congestion-eliminating investment of the firm that ties the service induces firms to compete more aggressively. The reason is that a more attractive service yields a significant reduction of the nonservicing firm’s market share. On the contrary, this is not the solution when the investment cost is high. Thus, both private and social outcomes result in not investing in eliminating the congestion.

Proposition 12 states that when the congestion rate \( (c) \) is high, having a social planner coordinating firms and a low investment cost, all consumers purchase the product tied-service
from the firm that invests in eliminating the congestion. This implies that the uncongested service pays the investment cost only one time. Such a coordination is not possible in the private outcome, so firms do not invest and they share equally the market. With high investment costs, the private and the social outcomes coincide.

<table>
<thead>
<tr>
<th></th>
<th>$c &lt; 1$</th>
<th>Non Regulated</th>
<th>Regulated</th>
<th>$c &gt; 1$</th>
<th>Non Regulated</th>
<th>Regulated</th>
<th>$\gamma &lt; \frac{1}{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_1$</td>
<td>$\frac{2e}{3(1+c)}$</td>
<td>$&lt; \frac{1}{2c+1}$</td>
<td>$\neq 0$</td>
<td>$D_1$</td>
<td>$\frac{1}{2}$</td>
<td>$\neq (0,c)$</td>
<td>$\gamma &gt; \frac{1}{2}$</td>
</tr>
<tr>
<td>$c_1$</td>
<td>$c$</td>
<td>$\gamma &gt; \frac{1}{2c+1}$</td>
<td>$= c$</td>
<td>$c_1$</td>
<td>$\left(c_1, c_2\right)$</td>
<td>$\gamma &gt; \frac{1}{2}$</td>
<td>$\left(c, c\right)$</td>
</tr>
</tbody>
</table>

Table 1. Comparison between the equilibrium and the social outcomes.

5 The optimal regulated price outcome

In this section we compare the private and the optimal regulated price outcome. Firstly, as in the previous section, we compare the investment decisions and the serviced product’s market shares. According to Proposition 10, if the congestion rate is low, that is, when $c < 1$, we compare the asymmetric private outcome in (4) with the asymmetric regulated price outcome in (10) and (11). If the congestion rate is high, that is, when $c > 1$, we compare the symmetric private outcome in (14) with the symmetric regulated price outcome in (19) and (20) (see Table 2).

**Proposition 13** When one firm ties-in the service while the other sells the product with no service, that is, when $c < 1$, we obtain the same result as in Proposition 11.

**Proposition 14** When both firms ties-in the service, that is, when $c > 1$, we have the following:

1. If the eliminating-investment cost rate is low, that is, when $\gamma < \frac{1}{2}$, the equilibrium and the social optimal congestion-eliminating investments do not coincide.
2. If the eliminating-investment cost rate is high, that is, when $\gamma > \frac{1}{4}$, the equilibrium congestion-eliminating investment decision coincides with the socially optimal decision.

Comparing propositions 11 and 12 with propositions 13 and 14, we observe that we obtain the same qualitative results. Therefore, the same intuitions apply in this section with respect to investment decisions and market shares.

Next, we compare the equilibrium and regulated prices (see Figures 1 and 2). In the competitive market, there is insufficient investment. We find that the social planner should set a regulated price either below/above the equilibrium price to induce the efficient investment, depending on the tie-in decision. The intuition is that tie-in decisions influence the degree of price competition in the private market.

**Proposition 15** When one firm ties-in the service while the other sells the product without service, that is, when $c < 1$, we have

1. For $0 < c < (-1 + \sqrt{3})/2$, the equilibrium prices are below the regulated prices, that is, when $\gamma < \frac{1}{2c+1}$, $p_N^* < p_S^* < \overline{p}_N = \overline{p}_S$, otherwise, $p_N^* < p_S^* < \overline{p}_N < \overline{p}_S$.

2. For $(-1 + \sqrt{3})/2 < c < (-1 + \sqrt{33})/8$, we have that, when $\gamma < \frac{1}{2c+1}$, $p_N^* < \overline{p}_S = \overline{p}_N < p_S^*$, otherwise, $p_N^* < \overline{p}_N < p_S^* < \overline{p}_S$.

3. For $(-1 + \sqrt{33})/8 < c < 1$, we have that, when $\gamma < \frac{1}{2c+1}$, $\overline{p}_N = p_S^* < p_N^* < p_S^*$, otherwise, $\overline{p}_N < p_N^* < p_S^* < \overline{p}_S$.

**Proposition 16** When both firm ties-in the service, that is, when $c > 1$, the equilibrium price is over the optimal regulated price, that is, $\overline{p}^+ < p^*$.

Propositions 15 and 16 indicates that the government should set a regulated price above (below) the equilibrium price when the congestion rate ($c$) is low (high). When the congestion is low, the nonservicing firm has a strong incentive to cut prices in the private market. This incentive is sufficiently strong to force low equilibrium prices. Thus, when price regulation exists, the social planner induces sufficient investment and serviced product market share by setting a regulated price above the equilibrium price. The contrary applies when the
congestion is high. The incentive to increase the market share is low. So, firms set high prices to extract the maximum consumer surplus towards service-oriented consumers. In this situation, when the price is regulated, the social planner can induce sufficient investment by setting a regulated price below the price equilibrium. At the equilibrium prices, firms would invest in regulated markets with a high investment cost.

<table>
<thead>
<tr>
<th></th>
<th>$c &lt; \frac{1}{2c+1}$</th>
<th>$c &gt; \frac{1}{2c+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma &lt; \frac{1}{2c+1}$</td>
<td>$D_S$</td>
<td>$\gamma &lt; \frac{1}{4}$</td>
</tr>
<tr>
<td>$c_S$</td>
<td>$\frac{2c}{3(1+c)}$</td>
<td>$\neq 0$</td>
</tr>
<tr>
<td>$(p_S, p_N)$</td>
<td>$\left(\frac{2c+1+2c}{3}, \frac{3}{3}\right)$</td>
<td>$\neq$</td>
</tr>
<tr>
<td>$\gamma &gt; \frac{1}{2c+1}$</td>
<td>$D_S$</td>
<td>$\gamma &gt; \frac{1}{4}$</td>
</tr>
<tr>
<td>$c_S$</td>
<td>$\frac{2c}{3(1+c)}$</td>
<td>$&lt; \frac{1}{2c+1}$</td>
</tr>
<tr>
<td>$(p_S, p_N)$</td>
<td>$\left(\frac{2c+1+2c}{3}, \frac{3}{3}\right)$</td>
<td>$= c$</td>
</tr>
</tbody>
</table>

Table 2. Comparison between the equilibrium and the regulated outcomes.
Figure 1. Equilibrium and regulated optimal prices when $\gamma < 1/(2c + 1)$.

Figure 2. Equilibrium and optimal regulated prices when $\gamma > 1/(2c + 1)$

6 Conclusions

We have shown that the introduction of congestion externalities in the tying model may have some significant impact on the firms’ tying decision. In our model, private firms take two different decisions: to tie a service and to invest in eliminating the congestion of the service. It is known that these strategic variables have contrary effects on price competition. The main effect of tying is that relaxes price competition when one firm ties the service and the rival does not. The reason for this is that firms use this strategy in order to differentiate themselves from the competing brands. Contrary, the main effect of investing in eliminating the congestion of the service is that makes price competition stronger. This is because when
congestion effects are at work, consumers’ congestion makes the demand addressed to each firm less elastic, thus reducing the incentive to lower the price. Although, in both cases, firms renounce to extract a part of the consumer surplus. On one hand, when firms use tying as a product differentiation strategy, one of them offers the basic product, which is less valued than a product tied to a service. On the other hand, if firms do not invest in eliminating congestion in the use of the service, the (indirect) utility of consumers when buying the tied product is lower. This all suggests that a key parameter in our analysis must be the degree of congestion (c).

We have shown that the introduction of congestion externalities towards the tying model have a significant impact on the market outcome. The main purpose of congestion is to slow down price competition, in order to change the profitability of tying as a product differentiation tactic. No firm undertakes congestion-eliminating investment. Thus, on the one hand, if the consumers’ sensibility for a congested service (c) is sufficiently low, firms use tying tactics in order to differentiate themselves from the competing brands. In this case, firms take decisions in order to minimize price competition. Thus, allowing the equilibrium outcome to prevent firms from extracting the maximum consumer surplus. Interestingly, on the other hand, tying a service to reduce price competition is not profitable when the consumers’ sensibility for a congested service is sufficiently high. Both firms offer a service to extract the higher possible consumer surplus. The reason is that, with the presence of congestion externalities, rivals compete less aggressively. If the rival cuts the price, that firm does not achieve a large enough increase in the market share to offset the decreased price. This is due to consumers with high sensibility that avoid firms with larger market shares.

It is also found that each firm may undertake insufficient investment from a social viewpoint. The congestion-eliminating investment cost (γ) will be determinant in this result. On one hand, when congestion is low, having higher sales of the serviced product is socially desirable; that is, the firm that ties the product with service underproduces from a social viewpoint. The servicing firm takes an advantage of the nonservicing firm, since consumers’ valuation for the serviced product is higher than for the basic product. Thus, the nonservicing firm cuts the price trying to maintain the market share. We have also shown that investment decisions do not coincide when the investment cost is low. A congestion-
eliminating investment from the product-service tying firm induces firms to compete more aggressively. The reason is that a more attractive service yields a significant reduction of the nonservicing firm’s market share. On the contrary, this is not the solution when the investment cost is high. Thus, both private and social outcomes result in not investing in eliminating the congestion. On the other, when the congestion rate is high, having a social planner coordinating firms and a low investment cost, all consumers purchase the product-tied service from the firm that invests in eliminating the congestion. This implies that the uncongested service pays the investment cost only one time. Such a coordination is not possible in the private outcome, therefore firms do not invest and they share equally the market. With high investment costs, the private and the social outcomes coincide.

Finally, we consider a model that incorporates price regulation. In many countries, the prices of medical services, telecommunications, etc. are highly regulated. We find that the social planner should set a regulated price above (below) the equilibrium price when the congestion is low (high). We also find that the social planner must set a regulated price below the competitive price to induce the first-best level of investment.

References


